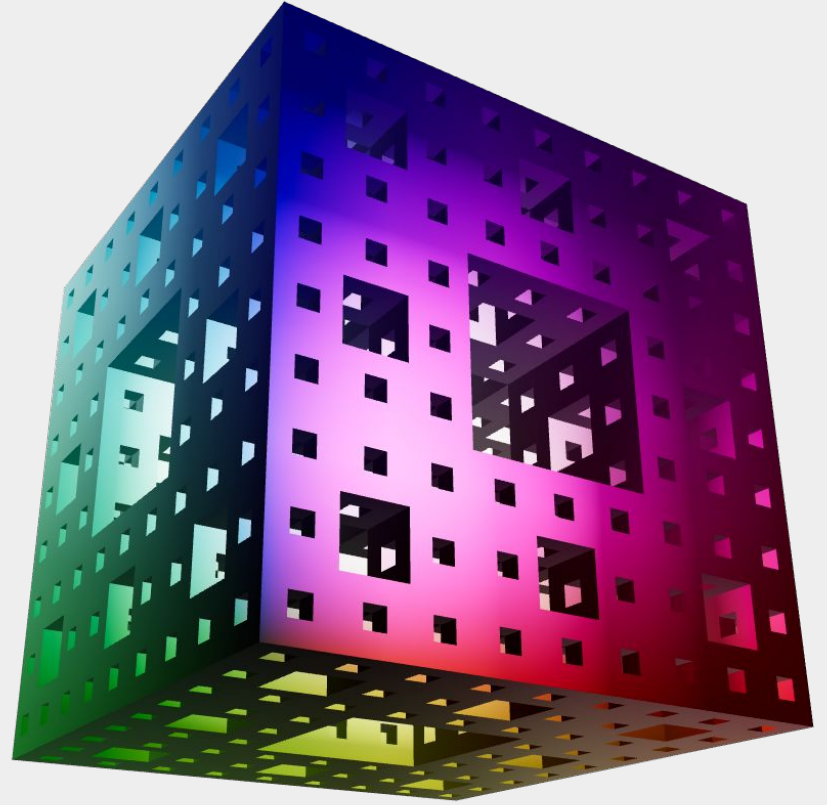


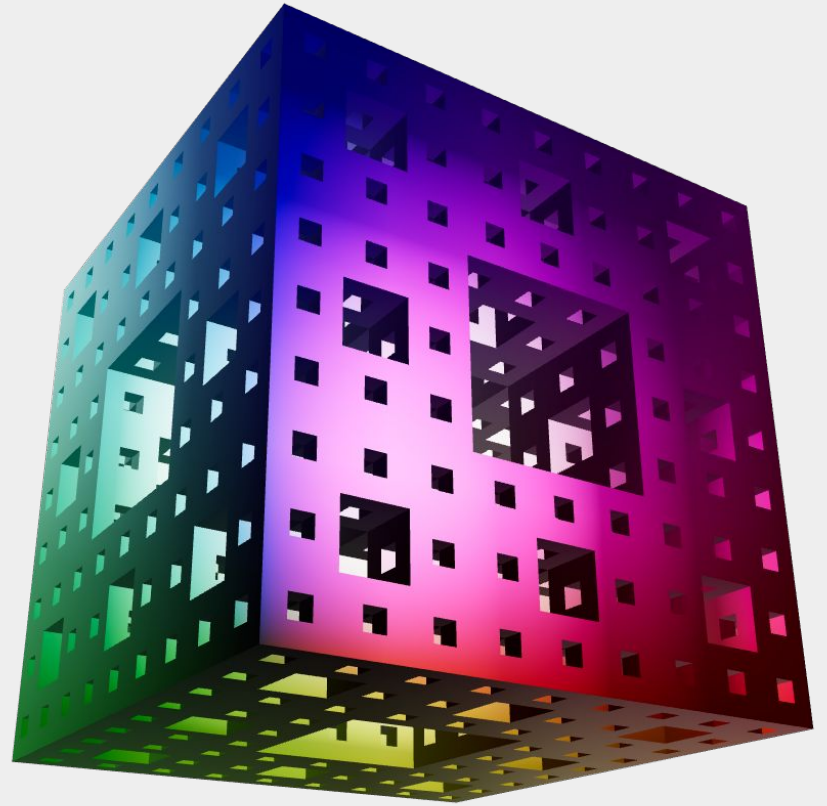
# Optimizing Mesh Booleans by Being Lazy (Experience Report)

Chun Kit Lam (HKUST)



~~Optimizing Mesh  
Booleans by Being Lazy  
(Experience Report)~~

*Optimizing a CG Library  
without learning CG*



master 14 Branches 28 Tags Go to file Code

elish	fix camera radius reset (#1733) ✓	2c98e8b · 2 days ago	1,559 Commits
.github	Add MANIFOLD_NO_IOSTREAM build-time option (#1690)	3 weeks ago	
.vscode	Some partial fixes for normals (#1716)	last week	
bindings	fix camera radius reset (#1733)	2 days ago	
cmake	Add CrossSection backend selector (#1710)	2 weeks ago	
docs	Whoops - looks like Doxyfile needs to be top level (#965)	2 years ago	
extras	Fix MSVC strict conversion warnings (#1726)	3 days ago	
include/manifold	Thread ExecutionContext into MeshGL ingest via Executio...	3 days ago	
samples	Deprecate compose (#1489)	4 months ago	
scripts	v3.5 (#1730)	3 days ago	
src	Thread ExecutionContext into MeshGL ingest via Executio...	3 days ago	
test	Thread ExecutionContext into MeshGL ingest via Executio...	3 days ago	
manifold	added MANIFOLD_ASSERT (#1225)	last year	
.codecov.yml	Changed MeshRelation API (#184)	4 years ago	
.gersemirc	format cmake files (#975)	2 years ago	
.gitattributes	Add MANIFOLD_NO_IOSTREAM build-time option (#1690)	3 weeks ago	
.gitignore	Updated Java bindings (#1617)	2 months ago	
AUTHORS	Add Geoff deRosenroll to author list (#351)	3 years ago	
CMakeLists.txt	v3.5 (#1730)	3 days ago	
CONTRIBUTING.md	Remove CLA requirement from CONTRIBUTING.md (#144...	6 months ago	
Doxyfile	fix docs and binding defaults (#1729)	3 days ago	
LICENSE	Initial commit	7 years ago	

About

Geometry library for topological robustness

- Readme
- Apache-2.0 license
- Contributing
- Activity
- 2.1k stars
- 35 watching
- 217 forks
- Report repository

Releases 23

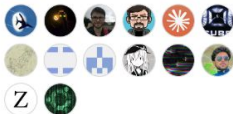
v3.5.0 (Latest) 3 days ago

+ 22 releases

Packages

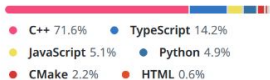
No packages published

Contributors 84



+ 70 contributors

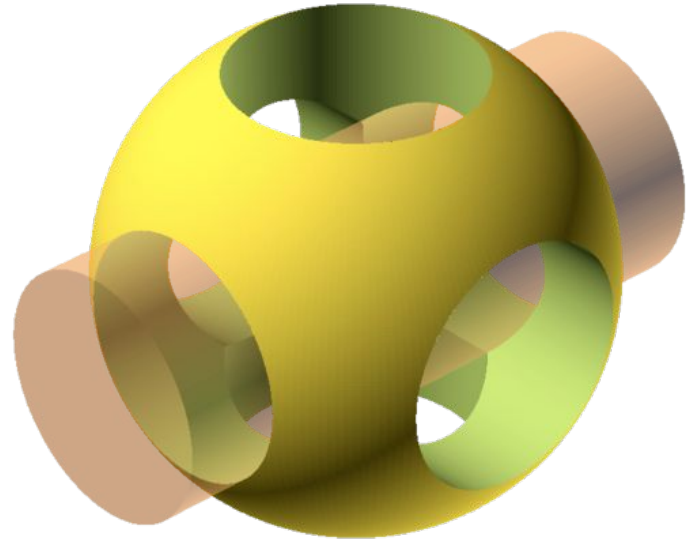
Languages



# Background: OpenSCAD

- Functional Language (*not a great one...*)
- Constructive Solid Geometry (CSG)

```
difference() {  
  sphere(d=50);  
  cylinder(d=25, h=65, ...);  
  rotate([90, 0, 0])  
    cylinder(...);  
  rotate([0, 90, 0])  
    cylinder(...);  
}
```



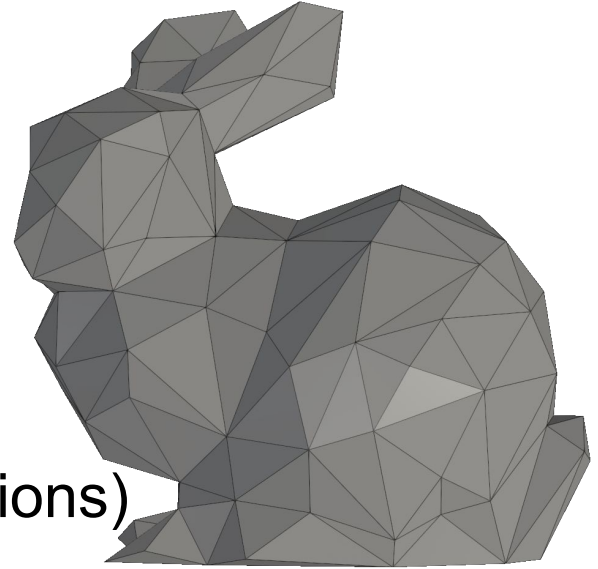
# Problem: OpenSCAD is *SLOW*...

- Compile time:  
minutes/hours
- Unacceptable for me:  
Need fast iterations
- More than 100x faster  
with manifold



# Why OpenSCAD is slow?

- Solid mesh modeling:  
More than just triangle soup
- Topo: Closed 2-manifold
- Geometry: No self-intersections
- Float: Robustness issues  
(Violates Euclidean Geometry Assumptions)
- Exact: Slow, Snap-Rounding



Model credit: <https://www.thingiverse.com/thing:151081>

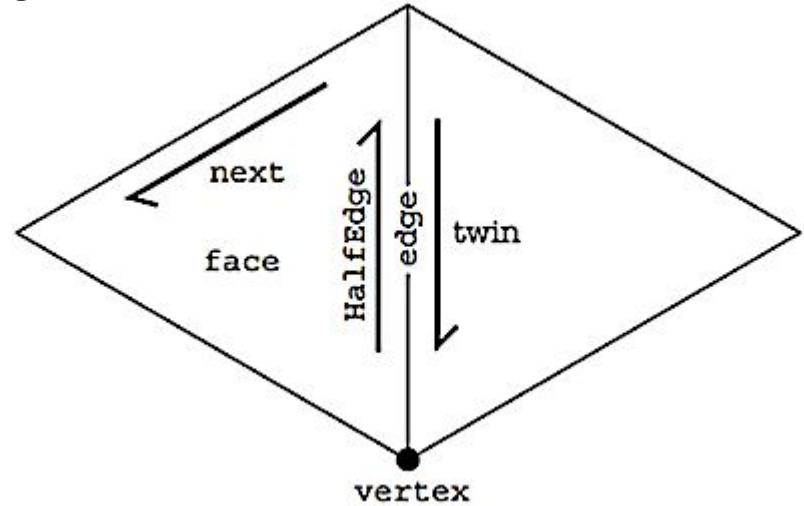
# How manifold solves this?

Implements Smith's robust inexact algorithm

- Careful floating point predicates
- Guaranteed topology:  
Even for random coords input

Topology over geometry correctness

- Broken topology is harder to fix
- Some algo/data structure relies on this
- Geometrically incorrect models are everywhere



# Manifold's API

Lazy:

- Copy (just shared)
- Booleans
- Affine Transforms

Triggers Eval:

- Queries

Simple, intuitive

No other libs have this (AFAIK)

Boolean Operations		
$\cup, -, \cap$	$M \rightarrow M \rightarrow M$	Mesh Booleans
Transformations		
translate, scale, rotate	$M \rightarrow \mathbb{R}^3 \rightarrow M$	Common Affine Transforms
transform	$M \rightarrow \mathbb{R}^{3 \times 4} \rightarrow M$	Affine Transform
warp	$M \rightarrow (\mathbb{R}^3 \rightarrow \mathbb{R}^3) \rightarrow M$	Vertex Transform
Constructors		
from_mesh	Mesh $\rightarrow M$	Construct a Manifold mesh object from a raw mesh
sphere	$\mathbb{R} \rightarrow M$	Sphere constructor
cube	$\mathbb{R}^3 \rightarrow M$	Rectangular cuboid constructor
Miscellaneous		
to_mesh	$M \rightarrow \text{Mesh}$	Get mesh output
decompose	$M \rightarrow \text{List } M$	Decompose the mesh into disjoint components.
surface_area	$M \rightarrow \mathbb{R}$	Surface area of the mesh
volume	$M \rightarrow \mathbb{R}$	Volume of the mesh
genus	$M \rightarrow \mathbb{N}$	Genus of the mesh

# Notes about Correctness

- Mesh Boolean not associative/commutative
- Rewrite changes results  
Like floating point rewrites
- Users should not depend on exact results
- Only guarantee determinism

$$A + (B + C)$$

#

$$(A + B) + C$$

#

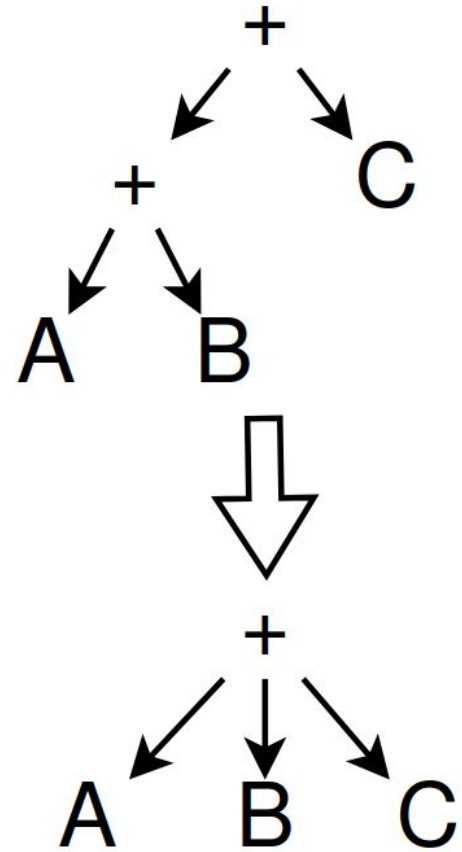
$$(B + A) + C$$

# AST Rewriting

- Goal: Maximize arity
  - Use optimized variadic Boolean
1. Propagate transforms to leaves  
 $t(a \cup b) = (t a) \cup (t b)$
  2. Maintain a normal form

$$\bigcirc \overline{n_1} - \overline{n_2} = \bigcirc n - \bigcup_{m \in \overline{n_2}} m$$

$$\bigcirc \in \{\cup, \cap\}$$



$$\boxed{\llbracket m \rrbracket_t = n}$$

ATOM

$$\overline{\llbracket p \rrbracket_t = \cup (t p)}$$

TRANSFORM

$$\overline{\llbracket t_0 m \rrbracket_t = \llbracket m \rrbracket_{t \times t_0}}$$

SUBTRACTION-1

$$\frac{\llbracket m_0 \rrbracket_t = \bigcirc \overline{n_0} - \overline{n_0}' \quad \llbracket m_1 \rrbracket_t = \cup \overline{n_1}}{\llbracket m_0 - m_1 \rrbracket_t = \bigcirc \overline{n_0} - (\overline{n_0}', \overline{n_1})}$$

SUBTRACTION-2

$$\frac{\llbracket m_0 \rrbracket_t = \bigcirc \overline{n_0} - \overline{n_0}'}{\llbracket m_0 - m_1 \rrbracket_t = \bigcirc \overline{n_0} - (\overline{n_0}', \llbracket m_1 \rrbracket_t)}$$

VARIADIC-1

$$\frac{\llbracket m_0 \rrbracket_t = \bigcirc \overline{n_0} \quad \llbracket m_1 \rrbracket_t = \bigcirc \overline{n_1}}{\llbracket m_0 \bigcirc m_1 \rrbracket_t = \bigcirc (\overline{n_0}, \overline{n_1})}$$

VARIADIC-2

$$\frac{i \in \{0, 1\} \quad \llbracket m_i \rrbracket_t = \bigcirc \overline{n_i}}{\llbracket m_0 \bigcirc m_1 \rrbracket_t = \bigcirc (\overline{n_i}, \llbracket m_{1-i} \rrbracket_t)}$$

VARIADIC-3

$$\overline{\llbracket m_0 \bigcirc m_1 \rrbracket_t = \bigcirc (\llbracket m_0 \rrbracket_t, \llbracket m_1 \rrbracket_t)}$$

## Rewriting Examples

Flatten:

$$(a \cup b) \cup (c \cup d) = a \cup b \cup c \cup d$$

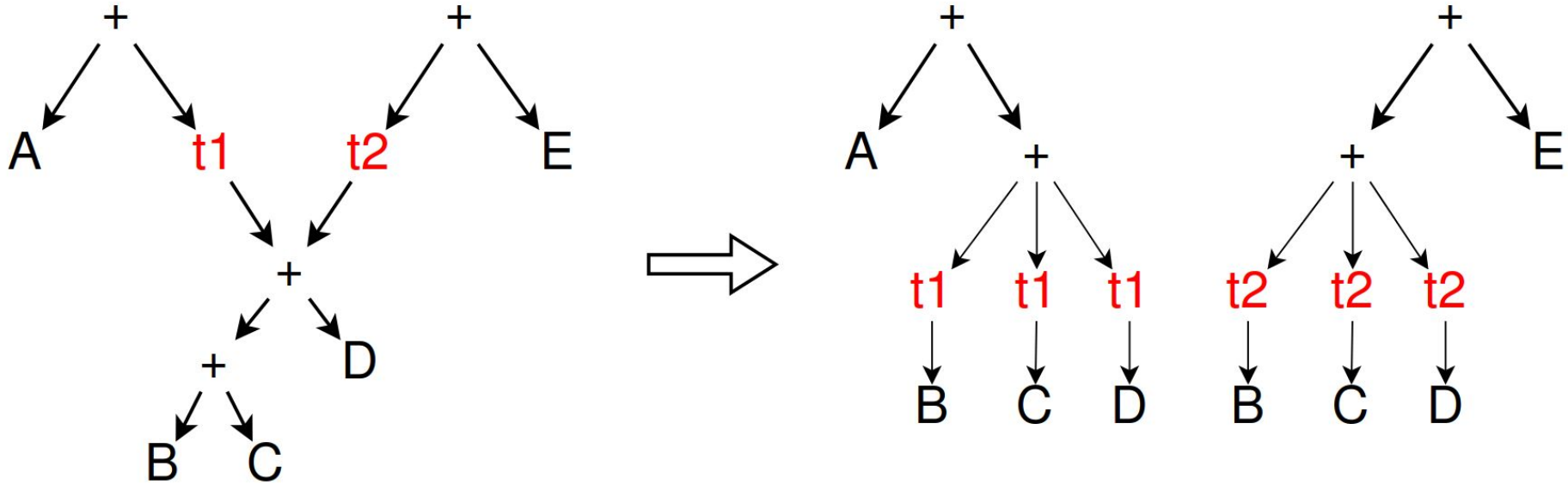
Negative operands:

$$(a - b) - (c \cup d) = a - (b \cup c \cup d)$$

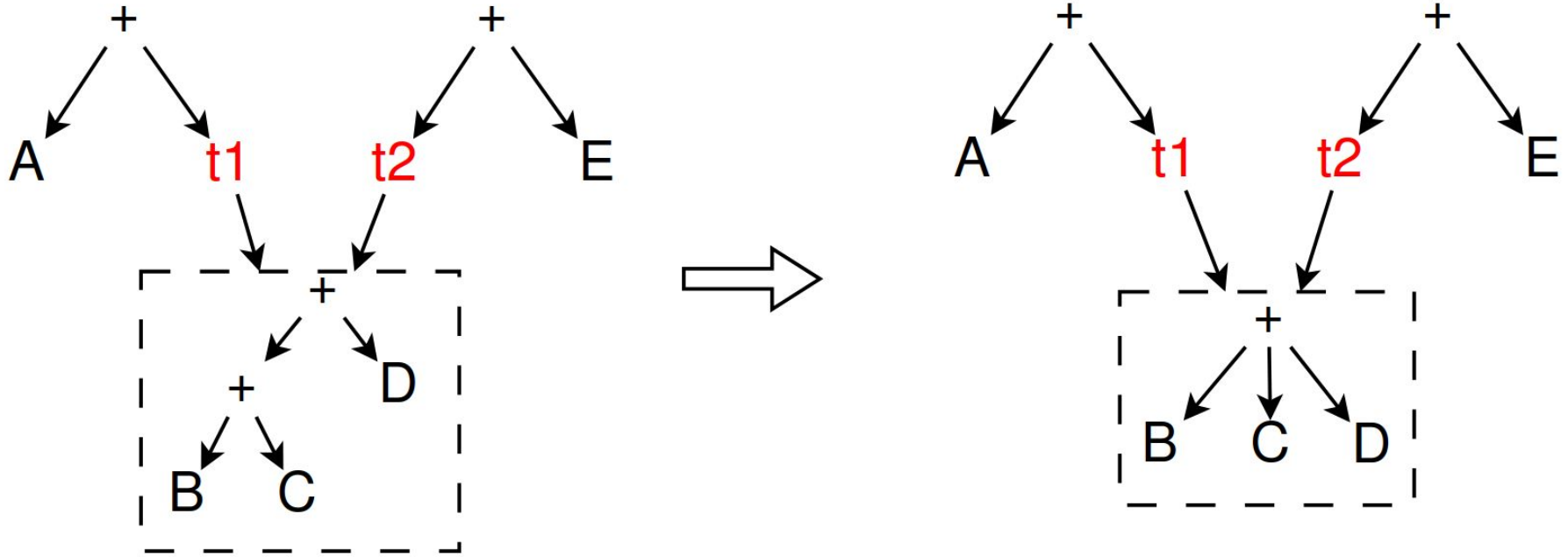
Positive operands:

$$(a - b) \cap (c \cap d) = (a \cap c \cap d) - b$$

# Shared Nodes and Work Duplication



# Shared Nodes and Work Duplication

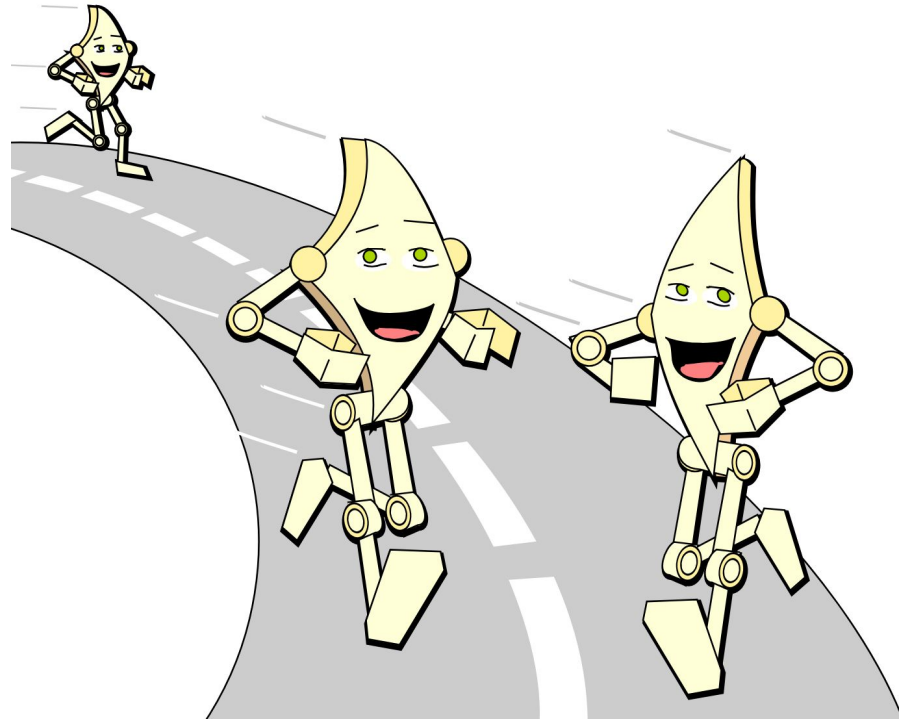


# Optimizing Variadic Booleans

1. Compose disjoint meshes
2. Smaller meshes first
3. Parallelize Booleans

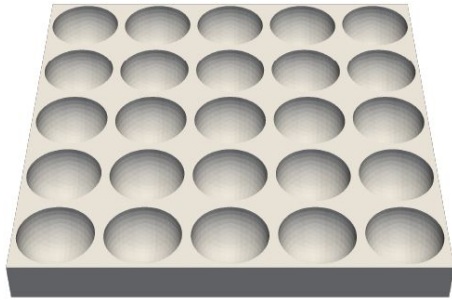
Naive implementation may  
cause data race!

Needs fixed-size batching

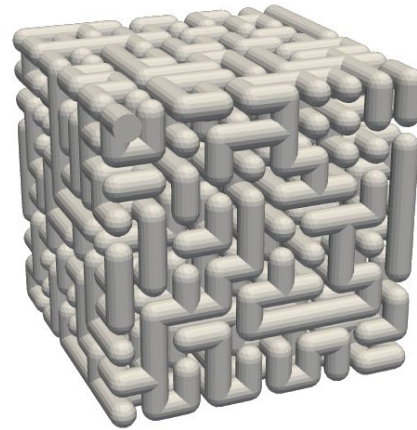


# Example

Two models from OpenSCAD PR #4533

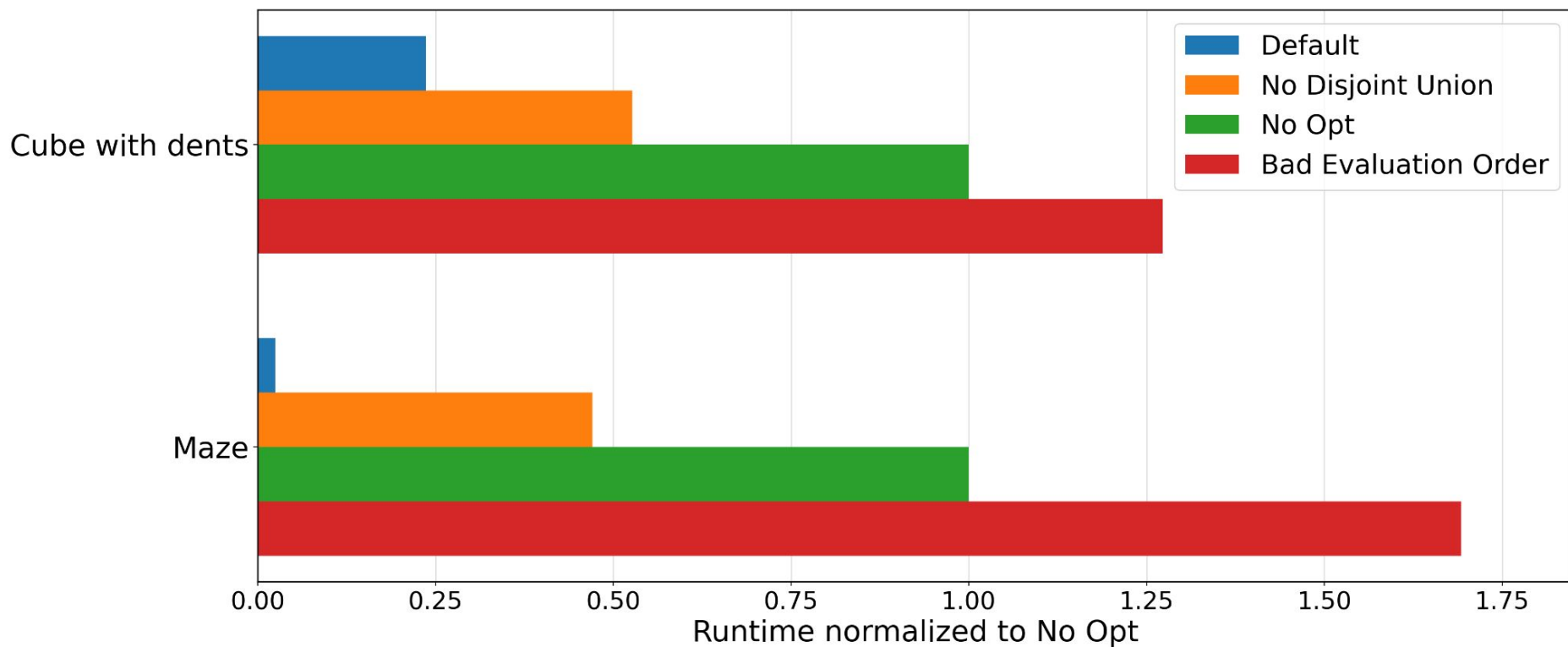


Cube with dents



Maze (negative part)

# Example



## Future Plan

- Dynamic rewriting:  
Subtraction vs Union
- More laziness internally,  
for better performance
- Support for  
constraint-based modeling
- More work on the  
geometry side...



<https://manifoldcad.org/>